TELECOMMUNICATION NETWORK RELIABILITY

S. Rai
Louisiana State University, Baton Rouge, USA

S. Soh
Curtin University of Technology, Perth, Western Australia

Keywords: Telecommunication network, terminal reliability, capacity related reliability, cutset, expected hop count, pathset, sum of disjoint products, Boolean technique, wireless communication network

Contents

Summary

Dependability analysis is an important parameter in the design of reliable telecommunication network. This chapter provides an overview of the telecommunication network reliability and presents two typical examples illustrating the concept and technique. First, various dependability measures, including terminal

reliability are discussed. Terminal reliability is defined as the probability that there exists one operative path between a given source and terminal pair nodes in the network. Second, pathset and cutset concepts and their enumeration methods are overviewed. Out of the various methods dealing with the evaluation of terminal reliability, the algorithms based on Boolean techniques are considered and discussed in details. The techniques use pathset and cutset information as sum-of products terms to generate mutually exclusive events. It is important to note that mutually exclusive terms have one-to-one correspondence with the probability expression. Further, Boolean techniques are generally efficient and produce compact expressions. The preprocessing of pathset and cutset and single- and multiple-variable inversion concepts, which help reduce the computing time for dependability measures, are discussed. In a general network, the reliability computation problem is NP-hard. To contrast this, special network structures where reliability computation is not too involved are also presented. Finally, two examples, namely capacity related reliability and expected hop count measures are described to illustrate the reliability measures and techniques. The terminal reliability assumes that the telecommunication network has equal link capacity and that link capacities are large enough to sustain the transmission of messages of any bandwidth. In practice, the capacity is a function of cost and is finite. Each link may have also different capacity. In addition to using link and/or node availability, capacity related reliability measure also considers this important parameter. Other example is from the wireless communication network, where again Boolean techniques are applied to compute the reliability constraints with expected hop count between a node pair.

1. Introduction

To illustrate the concept of telecommunication network reliability, consider a simple network with three centers, *X*, *Y*, *Z* as shown in Figure 1. Each of the centers communicates with each of the other centers by means of a 2-way link, where a link and/or a node may fail with certain probability. Centers *X* and *Y*, for example, communicate with each other via links *P* and *A*. Let us consider an application that requires each center to have a 2-way conversation with each of the other two centers, either directly or indirectly via the third center. In this example, there are five viable paths (pathset), namely, {*ABC*, *PQR*, *ABPQ*, *ACPR*, *BCQR*} to establish a 2-way conversation for a center with each of the other two centers. For the application, the system is considered *successful* if at least one of the five paths is operational, and the probability of the existence of such successful event defines a reliability criterion for the telecommunication network. The reliability measure assumes that network has equal link capacity or the link capacities are large enough to sustain the transmission of messages (packets) of any bandwidth (size). In practice, the link capacity is a function of cost and is limited. Thus, if link *C* has a capacity of 1 unit and all other links have 2 units capacity, paths *ABC*, *ACPR*, and *BCQR* will fail to provide a viable path assuming a requirement of 2-unit flow among communicating centers *X*, *Y*, and *Z*. Section 2.1, describing methods to compute reliability, uses Figure 1 to help illustrate the computation of a quantitative measure for network reliability. The capacity related reliability problem is discussed in Section 4.

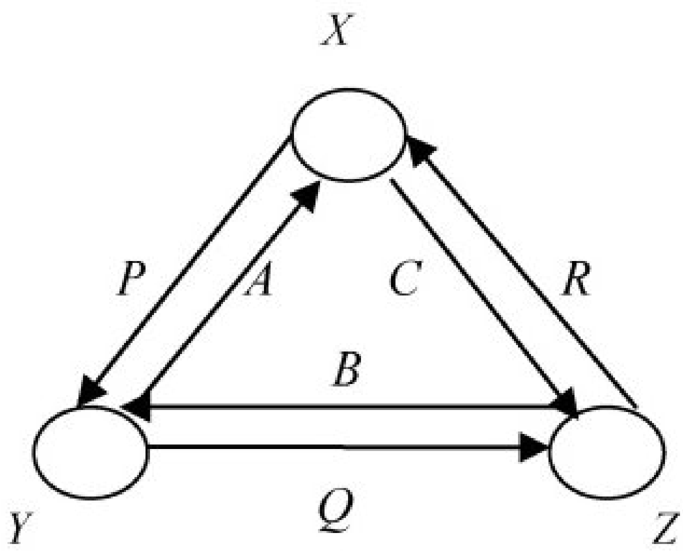

Figure 1: A 3-node telecommunication network

Using the notion in Figure 1, networks, in general, are used to interconnect multiple processing centers in distributed systems for several applications, such as telecommunications, the airline industry, and banking. Through resource sharing, a distributed processing system offers several advantages, such as increased performance, improved reliability of applications, and decreased costs. In designing such networks, a major consideration is often network reliability. This section presents various measures, models, assumptions and definitions related to the reliability studies in the telecommunication network. Each reliability measure is concerned with the ability of a network to be available to provide the desired service to the end users. In this sense, network reliability also refers to the steady state availability of a network. Further, dependability is also used as a synonym to reliability or availability. Since the focus of the chapter is telecommunication network reliability (and typical examples from the area), issues such as multimode and dependant-failure analysis, performability, and hardware and/or software reliability are not considered.

1.1 Measure

There exist a number of reliability measures depending on the system and network application. For telecommunication network, the focus is obviously on communication issues that meet certain connectivity requirements. For example, as an end user, it is important to identify the network's operational requirements. Perhaps the most common and natural operation in the network is *communication*; the telecommunication network reliability measures, studied in the literature, deal with connectivity and fall into one or more of the following four categories:

- *s-t* or 2-*terminal* reliability: the probability that a source node *s* communicates with a terminal node *t* for all node pairs
- *all-terminal* reliability: the probability that all operative nodes communicate
- *K-terminal* reliability or source-to-many terminals (SMT) reliability: the probability that a source node *s* communicates with some $K (K \geq 1)$ operative terminal nodes

- Many-sources-to-terminal (MST) reliability: the probability that some $K(K \geq 1)$ nodes communicate with a terminal node *t*

The network (modeled as a graph $G(V,E)$) reliability $R_K(G)$ for a set of specified nodes K is the probability that all elements of at least one minpath are working or one minus the probability that all elements of at least one mincut have failed. Note that the minpath, the mincut, and $R_K(G)$ depend on the specified node set K in $G(V,E)$ as well as the reliability measure under consideration. For $K = 2$, $R_K(G)$ represents 2-*terminal* reliability, and a minpath (mincut) is a simple path (cut) between an (*s*, *t*)-node pair. Section 1.3 provides the definitions of a simple path and a simple cut. For $K = V$, $R_K(G)$ represents *all-terminal* reliability, which is an important measure for reliable broadcasting in a telecommunication network. On the other hand, the SMT reliability is applicable for the resiliency issue of network multicasting, while the MST reliability has recently been used for analyzing the dependability of a distributed sensor network.

The 2-*terminal* reliability is defined as the probability that there exists at least one simple path between a source node *s* and a sink or terminal node *t*. This measure or its variance is most widely known in the literature. Besides, other measures like node-to-node grade of service (end-to-end blocking), average terminal reliability, functional reliability, etc., are also considered in the general framework of reliability problems. This chapter focuses on techniques to compute the 2-*terminal* reliability. Its variation, discussed in Section 4.1, considers the capacity related reliability, and is suitable for telecommunication networks. It uses two parameters, namely capacity of a link and the availability of the link, to compute the 2-terminal reliability of the network. Similarly, the reliability measure for a wireless communication network, discussed in Section 4.2, captures average hop count and node availability parameters together and reflects another variation of 2-terminal reliability.

1.2. Model and Assumptions

The dependability computation of a telecommunication network starts with its reliability model. One may choose a stochastic or a combinatorial model for a transient or a steady state network dependability analysis, respectively. Due to the nature of the problem considered in this chapter, the stochastic models based on the Markov or semi-Markov models are not discussed.

Various categories exist under combinatorial (also called non-state-space) model. Examples for the model include the reliability block diagram (RBD), fault trees (without or with repeated events), and reliability graphs. The RBD shows the functional relationships among resources and indicates which system elements must operate to accomplish the intended function successfully. A fault tree (FT) maps the operational dependency of a system on its components. However, unlike an RBD, the FT represents a probability of failure approach to system modeling. The phrase *without repeated events* means that inputs to all the gates are distinct, while *with repeated events* assumes non-distinct inputs. The reliability graph (RG) models the system as a graph, wherein each node is a computing unit (or processing entity) and links denote communication lines between them. For a telecommunication network, the reliability graph and the system graph could be used interchangeably. Nonetheless, the reliability graph has a probability of operation associated with each node and/or with each link. In this chapter, the reliability graph model is used to describe the various telecommunication network reliability issues.

The following basic assumptions are usually made for the network reliability analysis.

- All elements (nodes and/or links) of the network are always in active mode (no standby or switched redundancy).
- Each element in the network is represented as a two terminal device.
- The state of each element *i* of the network is either good (operating) with a probability p_i or bad (failed) with probability q_i; $p_i + q_i = 1$. Typically, the probability is obtained as $p_i = 1 - \frac{MTTR_i}{MTBF_i}$, where $MTTR_i$ ($MTBF_i$) refers to the mean time to repair (mean time between failures) of a component *i*.
- The states of all elements are statistically independent.
- The network is free from directed cycles and self-loops, as the success or failure of links in a directed cycle or self-loop do not alter the reliability computation.
- Note that these assumptions are helpful in making the reliability computation tractable.

1.3. Pathset and Cutset – Definition and Enumeration

In Section 1.1, various telecommunication network reliability measures have been defined. They need *s-t* pathset or cutset for computing *s-t* reliability. To define the structure functions minpath and mincut, let $w = (w_1, w_2, ..., w_n)$ be a state vector, where,

$$w_i = \begin{cases} 1, & \text{if component (node/link) } i \text{ is functioning} \\ 0, & \text{if component (node/link) } i \text{ has failed} \end{cases}$$

For a state vector *w*, a structure function, $\varphi(w)$, is defined by,

$$\varphi(w) = \begin{cases} 1, & \text{if system is functioning when the state vector is } w \\ 0, & \text{if system has failed when the state vector is } w \end{cases}$$

Systems for which $\varphi(w)$ is a nondecreasing function are called coherent systems. A *coherent system* can be represented by a network graph as well as by a fault tree without NOT gates. An *m*-out-of-*n* system with $n > 2$ and $1 < m < n$ is also an example of $\varphi(w)$ but it cannot be represented as a network (unless replicated links are allowed). In a network graph $G(V, E)$ with specified nodes $K \subseteq V$, if $K = \{s, t\}$, then a *minimal* or *simple s-t path (minpath)* is a vector w for which $\varphi(w) = 1$ and for all vectors $y < w$, $\varphi(y) = 0$. A vector $y < w$ if $y_i \le w_i$, $i = 1, 2, ..., n$, with $y_i < w_i$ for at least one i. A *pathset* is a set of all minpaths in the network. A *minimal* or *simple s-t cut (mincut)* is a vector w for which $\varphi(w) = 0$ and for all vectors $y > w$, $\varphi(w) = 1$. A *cutset* is a set of all mincuts in the network. A minproduct is a Boolean product term that represents a minpath or mincut of a coherent structure. This implies that a minproduct is a uniproduct, i.e., it contains only uncomplemented variables.

Example 1: In Figure 2, the *s-t* pathset contains four minimal paths *ab*, *cd*, *aed*, and *ceb*. Similarly, the cutset comprises mincuts ac, bd, aed, and *ceb.* Note that an *s-t* path helps connect *s-t* terminals, while an *s-t* cut interrupts the communication between the source and terminal nodes pair.

In a general network with k nodes and b links, there are approximately 2^{k-2} mincuts and 2^{b-k+2} minpaths (i.e, |cutset| = 2^{k-2} and |pathset| = 2^{b-k+2}). This means that the number of minimal paths and minimal cuts increases exponentially with an increasing network size (i.e., increasing number of nodes and/or links). Several pathset and cutset enumeration algorithms have been proposed in the literature. Most methods have considered a simplified way of obtaining the minimal paths or cuts, starting from either a connection matrix C, or with an incidence matrix of the network. Another approach employs a network decomposition strategy by enumerating simple paths between source and sink nodes, hence reducing the computational effort. The decomposition is typically performed through a minimal cut to partition the network into two subnetworks roughly in the center of the network. It has been shown that, among many such possible partitions, only one is optimum. Algebraic methods for generating mincuts in a network are also discussed in the literature. Several techniques in the literature are able to enumerate the pathset or cutset in linear time in the order of the number of minpaths or mincuts of a general network. Note that it is possible to convert a pathset into a cutset, and vice versa, by a process called *inversion*, which utilizes the two-step application of De Morgan's law. Some researchers have also described a computationally efficient method to obtain the pathset and cutset of a general network.

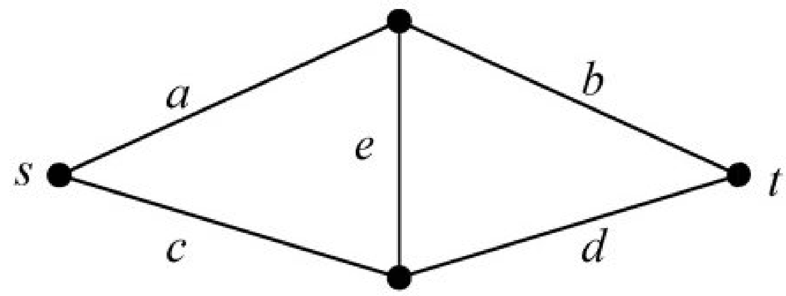

Figure 2: A bridge network

The time required to obtain all minpaths or mincuts is only a small fraction of the total time for the reliability evaluation problem, if the total number of minpaths ≥ 10. If the number of mincuts is smaller than the number of minpaths, then cutset-based procedures for calculating reliability would be more efficient. While the enumeration of mincuts is, in general, more complicated than is the enumeration of minpaths, there are several advantages to using them in network reliability analysis. It is possible to devise an exact algorithm for calculating the 2-*terminal* reliability of a directed graph $G(V, E)$ that is polynomial in the number of mincuts. It is, however, unlikely that a comparable algorithm exists that is polynomial in the number of minpaths. Thus, it has become imperative to enumerate both pathset and cutset and then to determine which approach to pursue. Another simple decision criterion regarding whether to pursue reliability or unreliability approach has been reported and requires only knowledge of the graph structure. The criterion states: if $b \geq 2k$, the number of minpaths is larger than the number of mincuts, and therefore the unreliability computation using cutset is more efficient.

A set of spanning trees for a graph model of a telecommunication network is useful for computing its *all-terminal* reliability. The set can also be used for *broadcast routing* in a store-and-forward packet-switching computer-communication network. Thus, it is important to look into a method to enumerate the minimum spanning trees for a given network. A spanning tree of a graph $G(V,E)$ is a subgraph that keeps all nodes of $G(V,E)$ connected and has no cycles. Many researchers have proposed sequential and parallel techniques to enumerate minimum weight or minimum spanning trees of a network. Note that a minimum spanning tree refers to a spanning tree for which the sum of the cost of the edges is minimal. Parallel and distributed algorithms do offer some speed advantages but are ideally suited for store-and-forward type packet switched computer networks.

2. Methods and Improving Computing Time

2.1. Methods

There exists a straightforward method based on *state enumeration* that helps compute the system reliability. To illustrate this, consider the 3-node telecommunication network shown in Figure 1. Here, one simply enumerates all states (that is all possible subgraphs), determines the success states (i.e., the states that include any of the five viable paths), and sums the occurrence probabilities of each of the success states. Table 1 shows the 18 possible success states for the example and each of their occurrence probabilities, where a p_i (q_i) denotes the success (failure) probability of a link *i*. (Assume that nodes are reliable.) Note that $p_i + q_i = 1$. Assuming equal probability of working, *p* for each link, the reliability expression (polynomial) for the system is given as *RN*= $p^6 + 6p^5(1-p) + 9p^4(1-p)^2 + 2p^3(1-p)^3$, and for *p*=0.9 the reliability measure for the example is *RN*=0.946242. Alternatively, the reliability problem can be solved by, first, determining all non-success states (i.e., the states that do not include any of the five viable paths). They are also shown in Table 1. The system unreliability, *QN*, is computed as the sums of the occurrence probabilities of each of the failure states (see Table 1), and *RN* = 1-*QN*. Using the probabilities of each failure states as shown in Table 1, the unreliability polynomial for the system is given as *QN*= $(1-p)^6 + 6p(1-p)^5 + 15p^2(1-p)^4 + 18p^3(1-p)^3 + 6p^4(1-p)^2$, and for *p*=0.9, *QN*=0.053758.

18 Success States					
State	Probability	State	Probability	State	Probability
ABC	$p_A p_B p_C q_P q_Q q_R$	*ACPR*	$p_A q_B p_C p_P q_Q p_R$	*ACPQR*	$p_A q_B p_C p_P p_Q p_R$
PQR	$p_P p_Q p_R q_A q_B q_C$	*APQR*	$p_A q_B q_C p_P p_Q p_R$	*ABPQR*	$p_A p_B q_C p_P p_Q p_R$

ABCP	$p_A p_B p_C p_P q_Q q_R$	*BCQR*	$q_A p_B p_C q_P p_Q p_R$	*BCPQR*	$q_A p_B p_C p_P p_Q p_R$
ABCQ	$p_A p_B p_C q_P p_Q q_R$	*BPQR*	$q_A p_B q_C p_P p_Q p_R$	*ABCPR*	$p_A p_B p_C p_P q_Q p_R$
ABCR	$p_A p_B p_C q_P q_Q p_R$	*CPQR*	$q_A q_B p_C p_P p_Q p_R$	*ABCQR*	$p_A p_B p_C q_P p_Q p_R$
ABPQ	$p_A p_B q_C p_P p_Q q_R$	*ABCPQ*	$p_A p_B p_C p_P p_Q q_R$	*ABCPQR*	$p_A p_B p_C p_P p_Q p_R$
46 Non-success States					
$\varnothing$	$q_A q_B q_C q_P q_Q q_R$	*CP*	$q_A q_B p_C p_P q_Q q_R$	*BCQ*	$q_A p_B p_C q_P p_Q q_R$
A	$p_A q_B q_C q_P q_Q q_R$	*CQ*	$q_A q_B p_C q_P p_Q q_R$	*BCR*	$q_A p_B p_C q_P q_Q p_R$
B	$q_A p_B q_C q_P q_Q q_R$	*CR*	$q_A q_B p_C q_P q_Q p_R$	*BPQ*	$q_A p_B q_C p_P p_Q q_R$
C	$q_A q_B p_C q_P q_Q q_R$	*PQ*	$q_A q_B q_C p_P p_Q q_R$	*BPR*	$q_A p_B q_C p_P q_Q p_R$
P	$q_A q_B q_C p_P q_Q q_R$	*PR*	$q_A q_B q_C p_P q_Q p_R$	*BQR*	$q_A p_B q_C q_P p_Q p_R$
Q	$q_A q_B q_C q_P p_Q q_R$	*QR*	$q_A q_B q_C q_P p_Q p_R$	*CPQ*	$q_A q_B p_C p_P p_Q q_R$
R	$q_A q_B q_C q_P q_Q p_R$	*ABP*	$p_A p_B q_C p_P q_Q q_R$	*CPR*	$q_A q_B p_C p_P q_Q p_R$
AB	$p_A p_B q_C q_P q_Q q_R$	*ABQ*	$p_A p_B q_C q_P p_Q q_R$	*CQR*	$q_A q_B p_C q_P p_Q p_R$
AC	$p_A q_B p_C q_P q_Q q_R$	*ABR*	$p_A p_B q_C q_P q_Q p_R$	*ABPR*	$p_A p_B q_C p_P q_Q p_R$
AP	$p_A q_B q_C p_P q_Q q_R$	*ACP*	$p_A q_B p_C p_P q_Q q_R$	*ABQR*	$p_A p_B q_C q_P p_Q p_R$
AQ	$p_A q_B q_C q_P p_Q q_R$	*ACQ*	$p_A q_B p_C q_P p_Q q_R$	*ACPQ*	$p_A q_B p_C p_P p_Q q_R$
AR	$p_A q_B q_C q_P q_Q p_R$	*ACR*	$p_A q_B p_C q_P q_Q p_R$	*ACQR*	$p_A q_B p_C q_P p_Q p_R$
BC	$q_A p_B p_C q_P q_Q q_R$	*APQ*	$p_A q_B q_C p_P p_Q q_R$	*BCPQ*	$q_A p_B p_C p_P p_Q q_R$
BP	$q_A p_B q_C p_P q_Q q_R$	*APR*	$p_A q_B q_C p_P q_Q p_R$	*BCPR*	$q_A p_B p_C p_P q_Q p_R$
BQ	$q_A p_B q_C q_P p_Q q_R$	*AQR*	$p_A q_B q_C q_P p_Q p_R$		
BR	$q_A p_B q_C q_P q_Q p_R$	*BCP*	$q_A p_B p_C p_P q_Q q_R$		

Table 1: The 2^6 states for the network in Figure 1

As is obvious, the state enumeration method requires the generation of all 2^M states for a system with *M* components, and is very inefficient. Thus, there exists a multiplicity of efficient methods for computing the 2-*terminal* reliability of a general network in the reliability literature. Figure 3 shows a classification of most reliability techniques.

The *approximate methods*, in general, are less complex than the exact methods and the closeness of the results largely depends on the correctness of the simplifying assumptions. Therefore, the major emphasis has been on the exact methods which require enumeration of minimal paths or cuts as an important step. *Exact reliability evaluation techniques* discussed in the literature fall into one of following three categories: *decomposition* or *factoring*, *inclusion-exclusion*, and *sum of disjoint products* (*SDP*). The factoring method concentrates on the operational and non-operational state of an individual link and is described as,

$$R(G) = p_e R(G \mid e \text{ is operational}) + (1 - p_e) R(G \mid e \text{ is not operational}), \tag{1}$$

where p_e is the probability that link e in graph $G(V, E)$ is functional. Note that Equation (1) uses the concept of Shannon's expansion principle in Boolean logic for reliability modeling. Several researchers have developed factoring algorithms that implement the series-parallel probability reductions and polygon reductions and use optimal link-selection strategies. It is important to note that this method can be employed using the graph representation, without knowing the connectivity information (i.e., minpath, mincut, or spanning tree).

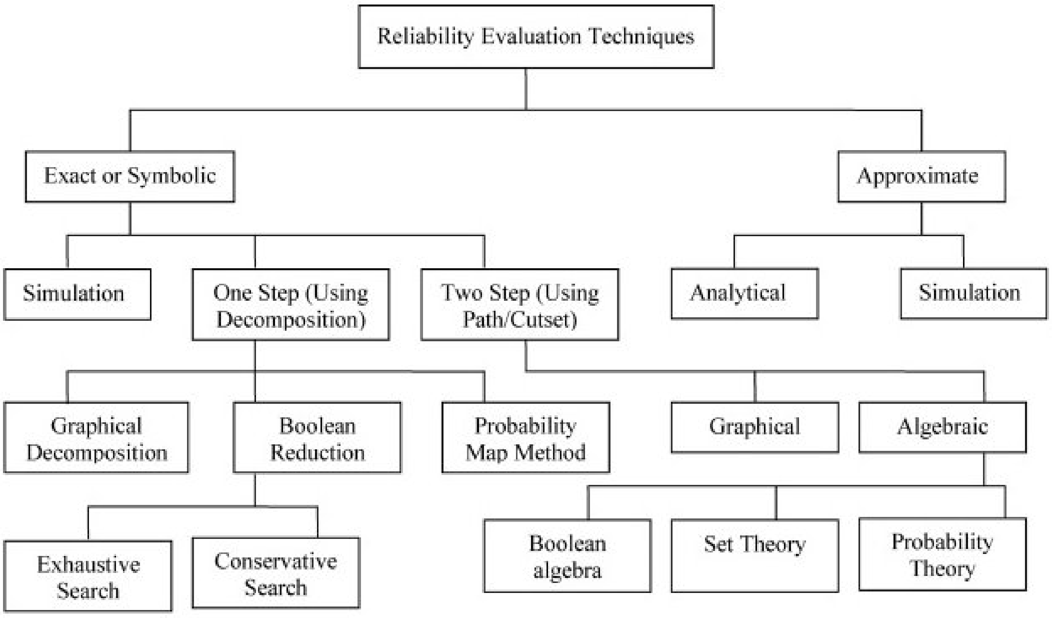

Figure 3: Classification scheme

By contrast, the inclusion-exclusion and sum of disjoint products techniques are based on a given enumeration of minpaths or mincuts. A straightforward application of the Poincaré formula for inclusion-exclusion generates $2^{\kappa}-1$ terms, where κ represents the number of minpaths or mincuts. Two different terms corresponding to the same event will cancel each other, however, if one is generated with odd formations and the other with even formations of minpaths. A formation is a set of minpaths whose union yields a subgraph containing s and t. This phenomenon was helpful to provide an algorithm based on the special form of the reduced inclusion-exclusion expression. Various combinatorial interpretations of the Poincaré formula is also given for $R_K(G)$.

To help understand the concept of sum of disjoint products (*SDP*) approach, let E_i be the event that all links in minpath MP_i operate. Reliability expression $R_K(G)$ is given by,

$$R_K(G) = P(E_1) + P(\overline{E_1}E_2) + \ldots + P(\overline{E_1}\,\overline{E_2}\ldots\overline{E_{\kappa-1}}E_\kappa)\,, \quad (2)$$

where κ is the total number of minpaths, $\overline{E}_i$ denotes the complement of event E_i, and $P(\cdot)$ is the probability function. Several algorithms based on the *SDP* formula in Equation (2) have been proposed. The *SDP* methods involve adding probabilities; however, the calculation of each constituent probability is, in general, quite involved. It is also important to emphasize that the efficacy of these methods can be highly dependent on the specific ordering given to the events E_i.

Example 2a: Consider the telecommunication network shown in Figure 1. The *SDP* technique requires the pathset of the system to generate its equivalent disjoint expression. A typical *SDP* approach generates a disjoint expression

$ABC + PQR\overline{ABC} + ABPQ\overline{C}\,\overline{R} + ACPR\overline{B}\,\overline{Q} + BCQR\overline{A}\,\overline{P}$ from the pathset {*ABC*, *PQR*, *ABPQ*, *ACPR*, *BCQR*}, where $\overline{X}$ denotes a component *X* is down. Note that a disjoint expression has one-to-one correspondence with the probability expression. Assuming *p* as the probability of working of each link, the reliability polynomial is given as *RN*= $p^3 + p^3(1-p^3) + 3p^4(1-p)^2$, which for *p*=0.9 yields *RN* as 0.946242. The quantitative measure is same as that obtained from the state enumeration method.

As explained earlier with the state enumeration approach, the reliability problem can alternatively be solved using a cutset concept as it captures the failure mechanism depicting 2-way conversation being not materialized. For this, any one of the six cuts (cutset), namely, {*AR*, *AQ*, *BP*, *BR*, *CP*, *CQ*} will disrupt the communication. Using the notion of negative logic, notation *AR* assumes that both links *A* and *R* are failed. An *SDP* technique generates a disjoint expression $AR + AQ\overline{R} + BP(\overline{A} + A\overline{R}\,\overline{Q}) + BR\overline{A}\,\overline{P} + CP\overline{B}(\overline{A} + A\overline{R}\,\overline{Q}) + CQ\overline{A}\,\overline{P}\,\overline{B}\,\overline{R}$ from the cutset, which is equivalent to an unreliability polynomial *QN*= $q^2 + 2q^2p + 3q^2p^2 + q^3p^2 + q^3p^3 - q^4p^2$. This gives *QN*=1 – *RN* = 0.053758 for *p*=0.9.

Example 2b: The simple paths for the bridge network given in Figure 2 are: *ab*, *cd*, *aed*, and *ceb*. An *SDP* technique (**E**-operator) gives a disjoint expression as $ab + (\overline{a} + a\overline{b})cd + aed(\overline{b}\,\overline{c}) + ceb(\overline{a}\,\overline{d})$. Thus, terminal reliability is $R(G) = p_a p_b + (q_a + p_a q_b) p_c p_d + p_a p_e p_d q_b q_c + p_c p_e p_b q_a q_d$. Note that an un-complemented (complemented) variable is replaced by availability (un-availability) term in the final expression. Further, the result shows a one-to-one correspondence between a Boolean *SDP* and a probability (reliability) expression for statistically independent random variable case. When $p_i = 0.9$ ($q_i = 0.1$) for all *i*, the network in Figure 2 has 2-*terminal* reliability metric $R(G) = 0.97848$.

Section 2.2, below, describes several methods for improving the computing time of the *SDP* algorithms, i.e, by preprocessing minpaths/mincuts, using multiple-variable inversion to reduce the number of Boolean products in *SDP* expressions, and by exploiting special structures, such as shellable systems or systems with redundancy structures. Three categories of *SDP* algorithms that use either single-variable or multiple-variable inversion are discussed in Section 3.

-
-
-

Bibliography

AboElFotoh H.M.F., Iyengar S.S., and Chakrabarty K., "Computing reliability and message delay for cooperative wireless distributed sensor networks subject to random failures," *IEEE Trans. Reliability*, vol. 54, no. 1, March 2005, pp. 145-155. [This paper describes a factoring-based technique to solve the reliability measures of wireless networks. It also shows that the reliability problems are #P.]

Balan A.O. and Traldi L. (2003), "Preprocessing Minpaths for sum of disjoint products," *IEEE Trans. Reliability*, vo. 52, no. 3, pp. 289-295. [This work proposes an optimal preprocessing technique for *SVI-SDP*.]

Barlow R.E. (1998), *Reliability Engineering*, Society for Industrial and Applied Mathematics, Philadelphia. [The book devotes to the analysis of failure data, optimal age replacement policies, system reliability, and Bayesian approach. It also describes influence diagrams, including fault trees.]

Colbourn C.J. (1987). *The Combinatorics of Network Reliability*, Oxford University Press, New York. [The book provides an excellent reading on reliability concepts, exact algorithms – both for general and restricted graphs, reliability polynomial and reliability bounds. Refer to this text for the details of factoring, inclusion exclusion, and *SDP* techniques]

Jane C-C. and Laih Y-W. (2004), "Algorithms to determine the threshold reliability of flow networks," *IIE Transactions*, vol. 36, pp. 469-479 [This paper describes a technique to compute *CRR* from *d*-path. It also provides a brief discussion on *d*-cut concept and its performance comparison with *d*-path concept.]

Lee S.M., Lee C.H., and Park D.H. (2004). Sequential capacity determination of subnetworks in network performance analysis. *IEEE Transactions on Reliability*, vol. 53, no. 4, pp. 481-486 [This paper presents an alternative algorithm to computing the capacity of an *h*-composite path (Equation 14).]

Rai S. and Agrawal D.P. (1990), *Distributed Computing Network Reliability*, IEEE Computer Society Press, Los Alamitos, California. [This tutorial text presents basic concepts in distributed computing network reliability. Covered topics such as pathset, cutset, and spanning tree enumeration, terminal reliability evaluation, constrained reliability measures, and reliability optimization provide an excellent reading material on the problem and techniques in reliability engineering.]

Rai S. and Agrawal D.P. (1990), Advances in Distributed System Reliability, IEEE Computer Society Press, Los Alamitos, California [This tutorial text presents advanced concepts in reliability theory. They include multimode and dependent failures, multiprocessor system reliability, and performability analysis.]

Rai S. and Soh S. (1991), "A computer approach for reliability analysis of large telecommunication-network with heterogeneous link-capacities," *IEEE Trans. on Reliability*, vol. 40, no. 4, pp. 441-451. [The described technique in Section 4.1.2 follows this paper. The paper also provides a survey on techniques to solve *CRR*. The key_cut and cross_link concepts to reduce the complexity of pathset-based technique are presented in the paper.]

Rai S., Veeraraghavan M., and Trivedi K.S. (1995), "A survey of efficient reliability computation using disjoint products approach," *Networks*, vol. 25, pp. 147-163. [This paper provides a general framework for various DSP techniques for reliability analysis.]

Shier D. R. (1991), *Network Reliability and Algebraic Structures*, Oxford University Press, New York. [This text book presents approaches, algebraic formulations, and bounds on reliability. It also provides pathset and cutset enumeration techniques and reliability covering problems.]

Shooman M.L. (2002), *Reliability of Computer Systems and Networks*, John Wiley & Sons, New York. [This book emphasizes fault tolerant issues in both hardware and software using static and dynamic redundancy. Networked systems reliability and reliability optimization are also discussed.]

Soh S. and Rai S. (1993), "Experimental results on preprocessing of path/cut terms in sum of disjoint products technique," *IEEE Trans. Reliability*, vol. 42, pp. 24-33. [This paper considers preprocessing techniques and experimentally evaluates their effect on computer time and number of terms generated in the final symbolic reliability expression.]

Soh S. and Rai S. (2005), "An efficient cutset approach for evaluating communication-network reliability with heterogeneous link-capacities," *IEEE Transactions on Reliability*, Vol. 54, No. 1, pp. 133-144 [This work provides literature survey and improved technique to solve capacity related reliability using cutset. The technique described in Section 4.1.3 follows this paper. The paper also presents an alternative algorithm that is more suitable for computing *CRR* of large telecommunication network.]

Soh S., Lim K-Y, and Rai S. (2006), "Evaluating communication network reliability with heterogeneous link-capacities using subset generation. *International Journal on Performability Engineering*, vol. 2, no. 1, pp. 3-17. [This paper presents an improved technique to generate subset cuts more efficiently.]

Soh S., Lau W, Rai S., Brooks, R.R. (2007), "On Computing Reliability and Expected Hop Count of Wireless Communication Networks. *International Journal on Performability Engineering*, vol. 3, no. 2, pp. 267-279. [This paper presents the SDP technique to compute the expected hop count of wireless communication network. Function NPG() in Section 4.2.1 is presented in this paper]

Biographical Sketches

Suresh Rai: Dr. Rai is a Professor with the Department of Electrical and Computer Engineering at Louisiana State University, Baton Rouge, Louisiana. Dr. Rai has taught and researched in the area of network traffic engineering, ATM, reliability engineering, fault diagnosis, neural net-based logic testing, wavelets, and parallel and distributed processing. He is a co-author of the book *Wave Shaping and Digital Circuits*; and tutorial texts *Distributed Computing Network Reliability*, and *Advances in Distributed System Reliability*. He has guest edited a special issue of IEEE Transactions on Reliability on the topic *Reliability of Parallel and Distributed Computing Networks*. He was an Associate Editor for *IEEE Transactions on Reliability* from 1990 to 2004. Currently, he is an editor for *International Journal of Performability Engineering*. Dr. Rai has published about 100 technical papers in the refereed journals and conference proceedings. He received the best paper award at the 1998 *IEEE International Performance, Computing, & Communication Conference* (Feb. 16-18, Tempe, Arizona; paper title: S. Rai and Y. C. Oh, Analyzing packetized voice and video traffic in an ATM multiplexer). Dr. Rai is a senior member of the IEEE.

Sieteng Soh received a B.S. degree in Electrical Engineering from University of Wisconsin-Madison, and M.S. & Ph.D. in Electrical Engineering from Louisiana State University, Baton Rouge. He was a faculty member (1993-2000), and the Director of the Research Institute (1998-2000) at Tarumanagara University-Indonesia. He is currently a Senior Lecturer with the Department of Computing at Curtin University of Technology, Perth, Western Australia. Dr. Soh has published several papers in some refereed international journals and conference proceedings in the area of computer network, network reliability, and parallel and distributed processing. He is a member of the IEEE.